Solutions to the European Kangaroo Pink Paper

1. **D** We first find the prime factorisation of $200013 - 2013$. We have $200013 - 2013 = 199000 = 198 \times 1000 = (2 \times 3^2 \times 11) \times (2^3 \times 5^3) = 2^4 \times 3^2 \times 5^3 \times 11$. From this we see that 2, 3, 5 and 11 are factors of $200013 - 2013$, but that 7 is not a factor.

2. **C** Each of the shaded regions is made by cutting rectangles out of the squares. When a rectangle is cut out of a corner it doesn't change the perimeter, but when a rectangle is cut out of an edge then the perimeter of the shaded region is greater than the original perimeter. Hence the perimeters of the first, fourth, fifth and sixth shapes are all equal in length to that of one of the squares, and those of the other two are greater.

3. **C** The prime factor decomposition of 1000 is $2^3 \times 5^3$ so the three numbers from the list must contain three factors of 2 and three factors of 5 between them. The only factors of 5 appear in $25 = 5^2$, $50 = 2 \times 5^2$ and $125 = 5^3$. The only way to obtain 5^3 from these is to use 125. This leaves 2^3 to be obtained from two other numbers, which can only be done using 2×4.

The sum of these numbers is $2 + 4 + 125 = 131$.

4. **E** Rewriting each number as a power of two, we get: $4^{15} = (2^2)^{15} = 2^{30}$ and $8^{10} = (2^3)^{10} = 2^{30}$. So the sum becomes $2^{30} + 2^{30} = 2 \times 2^{30} = 2^{31}$.

5. **E** The net of the cube consists of six large squares, each of which is split into four 2×2 squares. Each of these large squares must have 2 black squares and 2 white squares in alternating colours. This eliminates nets A, B, D.

Around each of the 8 vertices of the cube, there are either 3 black squares or 3 white squares. These squares must appear around the vertices in the net of the cube. This eliminates net C which has 2 squares of one colour, and one of the other colour around its vertices. And net E does indeed fold up to make the cube as required.

6. **C** The largest 3-digit multiple of 4 is 996, and the smallest is 100, so $4n - 4m = 996 - 100 = 896$.

7. **A** After rotation 90° anticlockwise, we obtain shape E. When reflected in the x-axis this gives shape A.

8. **A** The expressions can be rewritten as single square roots as follows:

A $20\sqrt{13} = \sqrt{400} \times \sqrt{13} = \sqrt{5200}$

B $\sqrt{20} \times \sqrt{13} = \sqrt{20 \times 13} = \sqrt{260}$

C $\sqrt{20} \times 13 = \sqrt{20} \times \sqrt{169} = \sqrt{3380}$

D $\sqrt{201} \times 3 = \sqrt{201} \times \sqrt{9} = \sqrt{201 \times 9} = \sqrt{1809}$

E $\sqrt{2013}$

It is then easy to see that A is the largest.

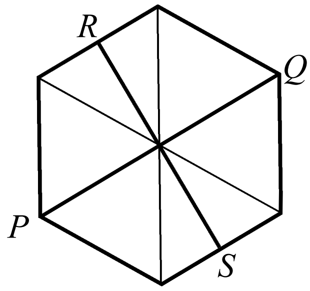
9. **D** Since triangle STR is equilateral, $\angle STR = 60°$. Hence $\angle STV = 130°$. Triangle STV is isosceles (since $ST = TV$), so $\angle TSV = \frac{1}{2}(180° - 130°) = 25°$. Thus $\angle RSV = 60° - 25° = 35°$.

10. **B** The two squares at either end of the shape contribute 3 cm towards the total perimeter of the zigzag. Each of the other 2011 squares contribute 2 cm towards the perimeter of the overall shape. Thus the perimeter of the zigzag is $2 \times 3 + 2011 \times 2 = 4028$ cm.

11. C The hexagon can be split into six congruent equilateral triangles. Each triangle has base of length $\frac{1}{2}PQ$ and height $\frac{1}{2}RS$, so the total area is

$$6 \times \tfrac{1}{2} \times \left(\tfrac{1}{2}PQ\right) \times \left(\tfrac{1}{2}RS\right) = \tfrac{3}{4} \times PQ \times RS = 60\,\text{cm}^2.$$

Hence $PQ \times RS = \frac{4}{3} \times 60 = 80$.



12. E Let T be the total number of points scored by the class, and let N be the number of students in the class. Let B be the number of boys.

Then the mean is $\frac{T}{N}$. If each boy scored an extra 3 points, this would increase T by $3B$, so the mean would be $\frac{T + 3B}{N} = \frac{T}{N} + \frac{3B}{N}$. This new mean would be 1.2 points higher than the original mean, so $\frac{3B}{N} = 1.2$, giving $\frac{B}{N} = 0.4$. But $\frac{B}{N}$ is the proportion of boys in the whole class, so the percentage of boys is 40%, leaving 60% girls.

13. A Since all the coordinates are negative, each of the calculations will yield a positive value. The smallest value will come from the least negative y-coordinate divided by the most negative x-coordinate; this comes from point A.

14. A The prime factorisation of 2013 is $3 \times 11 \times 61$. The factor pairs of 2013 are (1, 2013), (3, 671), (11, 183), and (33, 61). The only pair that could realistically be ages is 33 and 61. Hence John is 61 and was born in 1952.

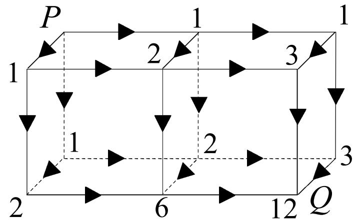
15. A The triangles are similar because they both contain angles of 59°, 60°, 61°. The smallest side of a triangle is always opposite the smallest angle, so line segment PR is the smallest edge of triangle PQR, though it is not the smallest edge of triangle PRS; hence triangle PQR is larger than triangle PRS and must contain the longest line segment. The longest side in a triangle is opposite the largest angle, so side PQ is the longest (opposite to $\angle PRQ$ which is 61°).

16. B Let the five consecutive integers be $n, n + 1, n + 2, n + 3, n + 4$. Ivana wants to split them into a pair and a triple with the same sum. First we show that $n + 4$ cannot be part of the triple. For if it were, then the triple would have a sum of at least $(n + 4) + n + (n + 1) = 3n + 5$, and the pair would have a sum at most $(n + 3) + (n + 2) = 2n + 5$. However, this is impossible since, if n is a positive integer, $2n + 5$ is less than $3n + 5$. Therefore the largest integer $n + 4$ must be in the pair. This gives four possible pairs.

Pair	Triple	Sums equal	Value of n
$(n+4)+n = 2n+4$	$(n+1)+(n+2)+(n+3) = 3n+6$	$2n+4 = 3n+6$	−2 (not positive)
$(n+4)+(n+1) = 2n+5$	$n+(n+2)+(n+3) = 3n+5$	$2n+5 = 3n+5$	0 (not positive)
$(n+4)+(n+2) = 2n+6$	$n+(n+1)+(n+3) = 3n+4$	$2n+6 = 3n+4$	2
$(n+4)+(n+3) = 2n+7$	$n+(n+1)+(n+2) = 3n+3$	$2n+7 = 3n+3$	4

There are only two sets of consecutive integers that can work, starting either with 2 or with 4.

17. D The arrows prevent any path from returning to a vertex already visited, so we can enumerate the number of different paths available to each vertex, beginning with the vertices nearest to P and working through to the vertex Q (shown on diagram). The number of paths to a particular vertex accumulate. In particular, Q can be reached from 3 vertices, which themselves can be reached in 3, 3, and 6 ways, so Q can be reached in $3 + 3 + 6 = 12$ ways.



18. D To turn the fraction into a decimal, we need to rewrite it with a denominator that is a power of ten: $\frac{1}{1024000} = \frac{1}{2^{10} \times 10^3} = \frac{5^{10}}{5^{10} \times 2^{10} \times 10^3} = \frac{5^{10}}{10^{10} \times 10^3} = 5^{10} \times 10^{-13}$ which has 13 decimal places. This is the least number of decimal places possible because 5^{10} is not divisible by 10.

19. E Let N be a number which is a multiple of 2013 and has exactly 2013 factors. We will show that N must have exactly three distinct prime factors. The prime factor decomposition of 2013 is $3 \times 11 \times 61$ so the prime factor decomposition of N must include powers of 3, 11, and 61; hence N certainly has at least three distinct primes in its prime factor decomposition.

Moreover, N cannot have more than three primes in its prime factorisation. To show this, it is useful to know that the number of factors of a number with prime factor decomposition $p_1^{r_1} \times p_2^{r_2} \times \ldots \times p_n^{r_n}$ is $(r_1 + 1)(r_2 + 1)\ldots(r_n + 1)$, where each $r_i \geqslant 1$, so each $(r_i + 1) > 1$. So for N to have 2013 factors, it is necessary for the product of these terms to be 2013. But the largest number of integers (greater than 1) that can multiply to make 2013 is three ($3 \times 11 \times 61$). Hence N can have at most three prime factors (3, 11, 61) and, as $(r_1 + 1)(r_2 + 1)(r_3 + 1) = 2013$, they appear with powers 2, 10, 60 in the prime factorisation of N.

Since the powers 2, 10, 60 can be assigned to the primes 3, 11 and 61 in six different ways, and each order yields a different integer N, there are 6 such possible values for N.

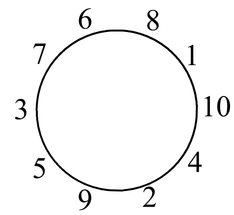
20. B Each fraction that Sylvie forms will have integer value if the denominator is a factor of the numerator. It is possible to find ten fractions that have integer values:

$$\frac{13}{1}, \frac{4}{2}, \frac{21}{3}, \frac{15}{5}, \frac{12}{6}, \frac{14}{7}, \frac{16}{8}, \frac{18}{9}, \frac{20}{10}, \frac{22}{11}.$$

It is not possible to make eleven integers. Since 13, 17, 19 are prime but do not have multiples on the list, they can appear only if they are on the numerator with 1 as the denominator. This can happen in only one of them, so the other two must form fractions without integer value (putting them together in the same fraction allows the ten integers listed above).

21. E If Julio starts with a set of three consecutive integers, $\{n - 1, n, n + 1\}$, then applying his procedure gives him $\{2n + 1, 2n, 2n - 1\}$ which, when put in order, is $\{2n - 1, 2n, 2n + 1\}$. That is, he obtains another set of three consecutive integers but with the middle number double that of the original set. Thus when he starts with $\{1, 2, 3\}$, the middle numbers that result from repeatedly applying his procedure are 2, 4, 8, 16, … , i.e. the powers of two. Since 2013 is not a power of two, nor is it one more or one less than a power of two, it will not appear in any set produced by Julio.

22. B It is possible for the ten new numbers to all be at least 15, and the diagram shows one way of achieving this (a little trial and improvement is required to obtain an example that works).



However, it is not possible for all the new numbers to be at least 16. For if it were possible, then we could split the original numbers 1, 2, …, 10 as follows:

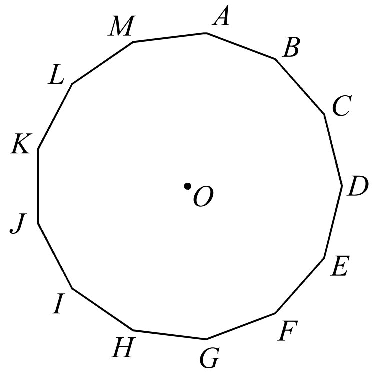
Going clockwise from the number 10, the next three numbers must add to at least 16.

The three numbers after that must add to at least 16, and the three numbers after that must add to at least 16.

When we add in the number 10 itself, we have a sum that must be at least 16 + 16 + 16 + 10 = 58, but we know that the numbers 1, 2, …, 10 add to 55. Hence it is not possible to achieve at least 16.

23. B Let n be the number of triangles with vertex O. Then the sum of the angles at the vertex O is $m + 2m + 3m + \dots + nm = (1 + 2 + 3 + \dots + n)m$ and must equal 360 (angles around a point). To minimise m we should find the largest value of n for which $(1 + 2 + 3 + \dots + n)$ is a factor of 360. Starting with $n = 1$, and increasing n by one each time until the sum exceeds 360, we get 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378. The largest one of these that is a factor of 360 is 120, which gives $m = 360 \div 120 = 3$ when $n = 15$.

24. D First consider the triangles that use vertex A. If the second vertex is B, then the only triangle that contains point O must use vertex H.



If the second vertex is C, then the third vertex could be H or I (2 triangles).

If the second vertex is D, then the third vertex could be H, I or J (3 triangles).

If the second vertex is E, then the third vertex could be H, I, J or K (4 triangles).

If the second vertex is F, then the third vertex could be H, I, J, K or L (5 triangles).

If the second vertex is G, then the third vertex could be H, I, J, K, L or M (6 triangles).

If the second vertex is H, I, J, K, L, M, then the third vertex would have to be one of B, C, D, E, F, G so these triangles would have been already counted above.

Hence the number of triangles which use vertex A are 1 + 2 + 3 + 4 + 5 + 6 = 21. Since there are thirteen possible vertices that we could have started with, we might expect 21×13 triangles. But each triangle uses three vertices so we have counted each triangle three times. Hence the number of triangles is $21 \times 13 \div 3 = 91$.

25. A Let d metres be the distance travelled by the tractor in the time it takes Yurko to walk a pace of one metre. When Yurko was walking in the same direction as the tractor, he moved a distance of $1 - d$ metres along the pipe with each pace. It took him 140 paces, so the length of the pipe is $140(1 - d)$ metres. When he is walking in the opposite direction, each pace moves him a distance of $1 + d$ metres along the pipe. It takes 20 paces, so the length of the pipe is $20(1 + d)$ metres. Hence we have $140(1 - d) = 20(1 + d)$, which gives $140 - 140d = 20 + 20d$, leading to $160d = 120$. Then $d = \frac{3}{4}$ so the length is $20\left(1 + \frac{3}{4}\right) = 35$ metres.